Version 1.0: 0608

ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MS04 Statistics 4

Mark Scheme

2008 examination - June series

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Dr Michael Cresswell Director General

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

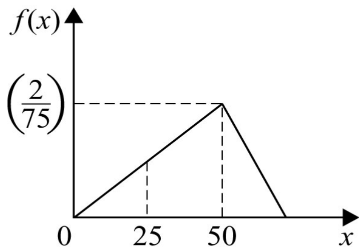
MS04

Q	Solution	Marks	Total	Comments
1(a)	$s = 976.09$	B1		
	$v = 9$	B1		
	$\chi_9^2(0.005) = 1.735$			
	$\chi_9^2(0.995) = 23.589$	B1		
	99% CL for σ are:			Or σ^2
	$\sqrt{\frac{9\times 976.09^2}{23.589}}$ and $\sqrt{\frac{9\times 976.09^2}{1.735}}$	M1 A1√		√ on s^2 and χ^2 (with or without $\sqrt{\ }$)
	99% CI is (603, 2220)	A1	6	AWRT
(b)	eg: Weather conditions Load Pilot	E1	1	Any sensible alternative
	Total		**7**	
2(a)	$\mathrm{E}(X) = p + 2pq + 3pq^2 + \ldots.$	M1		
	$= p\left(1 + 2q + 3q^2 + \ldots.\right)$	A1		
	$= \frac{p}{(1-q)^2}$			
	$= \frac{p}{p^2}$			
	$= \frac{1}{p}$	A1	3	AG (working required)
(b)(i)	6	B1	1	
(ii)	$\mathrm{P}(X \leqslant 6) = \frac{\left(\frac{1}{6}\right)\left(1-\left(\frac{5}{6}\right)^6\right)}{\left(1-\frac{5}{6}\right)}$	M1		
	$= 0.665$	A1		
	$\mathrm{P}(X > 6) = 0.335$	A1	3	$\left(\frac{5}{6}\right)^6 = 0.335$ B3 AWRT
(iii)	$1 - \left(\frac{5}{6}\right)^r > 0.9$	M1		
	$\Rightarrow \left(\frac{5}{6}\right)^r < 0.1$	A1		
	$\Rightarrow r > \frac{\log 0.1}{\log\left(\frac{5}{6}\right)}$	M1		
	$= 12.6$			
	$\therefore\ r = 13$	A1	4	CAO
	Total		**11**	

MS04 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\overline{x_1} = 192 \quad s_1 = 19.736$	B1		Both CAO
	$\overline{x_2} = 162 \quad s_2 = 16.362$	B1		Both (AWRT 19.7, 16.4)
	$s^2 = \dfrac{8\times19.736^2 + 7\times16.362^2}{8+7}$	M1		
	$= 332.\dot{6} \quad \left(=18.239^2\right)$	A1		AWRT 18.2
	$v = 15$	B1		
	$t = 2.131$	B1		AWRT 2.13
	$192 - 162 = 30$ $\therefore$ 95% CL are:			
	$30 \pm \left(2.13\times18.239\sqrt{\dfrac{1}{9}+\dfrac{1}{8}}\right)$	M1 A1√		√ on t and s^2
	95% CI is (11.1, 48.9)	A1	9	AWRT
(b)	$10 \notin$ CI	E1√		
	$\therefore$ reject claim	e1√	2	√ on (a)
	Total		**11**	
4(a)(i)	$\mathrm{F}(x) = 1 - \mathrm{e}^{-\frac{x}{200}}$			May be quoted
	$\mathrm{P}(X < 120) = 1 - \mathrm{e}^{-0.6}$	M1		
	$= 0.451$	A1	2	AWRT
(ii)	$\mathrm{P}(X > 160) = \mathrm{e}^{-0.8}$	M1		
	$= 0.449$	A1	2	AWRT
(iii)	$\mathrm{P}(X < 160 \mid X > 120)$			
	$= \dfrac{1 - [0.4512 + 0.4493]}{1 - 0.4512}$	M1		or $= \mathrm{P}(X < 40)$
		A1		$= 1 - \mathrm{e}^{-0.2}$
	$= 0.181$	A1	3	AWRT
(b)	$1 - \mathrm{e}^{-\frac{m}{200}} = 0.5$			
	$\Rightarrow \mathrm{e}^{-\frac{m}{200}} = 0.5$	M1		
	$\Rightarrow m = \ln 0.5 \times (-200)$	M1		
	$= 139$ hours	A1	3	AWRT
	Total		**10**	

MS04 (cont)

Q	Solution	Marks	Total	Comments
5(a)		B1 B1	2	Shape x–scale
(b)(i)	H_0 : triangular distribution fits	B1		
	Areas $\frac{1}{6}, \frac{1}{2}, \frac{1}{3}$	M1 A1		
	O_i 7 28 25 E_i 10 30 20	A1√		
	$\chi^2_{\text{calc}} = \frac{9}{10} + \frac{4}{30} + \frac{25}{20}$	M1		
	$= 2.28$	A1		Accept 2.25 to 2.30
	$v = 3 - 1 = 2$	B1		
	$\chi^2_{\text{crit}} = 5.991$	B1		AWRT 5.99
	$2.28 < 5.991 \Rightarrow$ Accept H_0 Triangular distribution fits data at 5% level of significance	A1√	9	
(ii)	$E_1 < 5 \Rightarrow$ combine classes $\left(\frac{1}{6} \times 15 = 2.5\right)$	M1		
	$v = 2 - 1 = 1$	A1	2	Or gives new χ^2_{calc}
	Total		**13**	

MS04 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$H_0 : \sigma^2 = 225 \quad H_1 : \sigma^2 \neq 225$	B1		Both
	$v = 15 - 1 = 14$	B1		
	$\left.\begin{array}{l}\chi^2_{14}(0.025) = 5.629 \\ \chi^2_{14}(0.975) = 26.119\end{array}\right\}$	B1		Both; or $F(\infty, 14) = 2.487$
	$\chi^2 = \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{14 \times 9.1^2}{225} = 5.15$	M1 A1		$F_{calc} = \dfrac{225}{9.1^2} = 2.72$
	$5.15 < 5.629 \Rightarrow$ Reject H_0			$2.72 > 2.487 \Rightarrow$ Reject H_0
	Evidence to suggest that variance is not 225	A1√	6	
(b)	$H_0 : \sigma_B^2 = \sigma_G^2 \quad H_1 : \sigma_B^2 \neq \sigma_G^2$	B1		Both
	$\left.\begin{array}{l}s_B^2 = 70.567 \\ s_G^2 = 14.25\end{array}\right\}$	B1		Both; or $s_B = 8.400 \quad s_G = 3.7749$
	$F_{calc} = \dfrac{70.567}{14.25} = 4.95$	M1 A1√		AWRT; √ on variances
	$v_1 = 5 \quad v_2 = 3$	B1		
	$F_{5,3} = 14.88$	B1		
	$4.952 < 14.88$ $\Rightarrow$ Accept H_0			
	Variances are equal	A1√	7	
	Total		**13**	

MS04 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$\sigma^2 = \mathrm{E}(X_i^{\,2}) - \mu^2$			
	$\Rightarrow \mathrm{E}(X_i^{\,2}) = \sigma^2 + \mu^2$	M1		
	$\mathrm{Var}(\bar{X}) = \mathrm{E}(\bar{X}^2) - \mu^2 = \dfrac{\sigma^2}{n}$	M1M1		
	$\Rightarrow \mathrm{E}(\bar{X}^2) = \dfrac{\sigma^2}{n} + \mu^2$		3	AG
(ii)	$nV = \sum_1^n X_i^{\,2} - n\bar{X}^2$			
	$\Rightarrow \mathrm{E}(nV) = \mathrm{E}\left\{\sum_1^n X_i^{\,2}\right\} - \mathrm{E}(n\bar{X}^2)$	M1		
	$= n(\sigma^2 + \mu^2) - (\sigma^2 + n\mu^2)$	M1		
	$= (n-1)\sigma^2$			
	$\Rightarrow \mathrm{E}\left(\dfrac{nV}{n-1}\right) = \sigma^2$	A1	3	
(b)	$\mathrm{E}(X) = \frac{1}{2}(X_1 + X_2)$			
	$V = \frac{1}{2}(X_1^2 + X_2^2) - \frac{1}{4}(X_1 + X_2)^2$	M1		or $\mathrm{E}\left[\frac{1}{2}(X_1 - X_2)^2\right]$
	$= \frac{1}{4}(X_1^2 - 2X_1X_2 + X_2^2)$			$= \frac{1}{2}\mathrm{E}(X_1^{\,2}) - \mathrm{E}(X_1X_2) + \frac{1}{2}\mathrm{E}(X_2^{\,2})$
	$= \frac{1}{4}(X_1 - X_2)^2$	A1		$= \mathrm{E}(X_1^{\,2}) - \{\mathrm{E}(X_1)\}^2$
	$\dfrac{nV}{n-1} = \dfrac{2}{1} \times \dfrac{(X_1 - X_2)^2}{4}$	M1		$= \sigma^2 + \mu^2 - \mu^2$
	$= \frac{1}{2}(X_1 - X_2)^2$	A1	4	$= \sigma^2 \Rightarrow$ unbiased
	Total		**10**	
	TOTAL		**75**	